Transient Flow and Mass Transfer of a Third Grade Fluid Past a Vertical Porous Plate in the Presence of Chemical Reaction

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Abstract

The effects of chemical reaction and suction, on the transient flow of an incompressible third grade fluid past a vertical porous plate, are studied. The governing time-based coupled partial differential equations are discretized using an efficient and unconditionally stable Crank-Nicolson finite difference scheme and the resulting algebraic nonlinear equations are solved by the modified Newton's method. The velocity and species concentration profiles are computed, graphically presented and discussed in detail for various values of embedded pertinent flow parameters. It is concluded that the fluid velocity decreases with increasing suction and viscoelastic second grade parameters while it increases with an increase in the Schmidt number, solutal Grashof number and third grade parameter. However, as the Schmidt number, suction and rate of chemical reaction parameters increase, they decrease the species concentration of the flow field.

Introduction

Concentration differences, applied external forces, chemical reaction and the medium, in which many transport processes exist in natural and technological applications, affect the mass and momentum transport. The study of mass transfer, with some physical variables, is useful for improving a number of chemical technologies such as polymer production, aerodynamic extrusion of plastic sheets, manufacturing of ceramics and cooling of nuclear reactors, e.t.c. The effect of homogeneous first-order chemical reaction in a laminar boundary layer flow in the neighbourhood of a horizontal flat plate for destructive and generative chemical reaction was investigated by Chambre and Young (1958). Apelblat (1980) studied the problem of mass transfer with chemical reaction of first-order. He proposed analytical solutions and asymptotic expressions for homogeneous and heterogeneous chemical reactions and gave some extensions for mass transfer in non-Newtonian liquids or in the case of permeable surfaces.

Das *et al.* (1994) examined the influence of first-order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. They employed an exact solution by using Laplace transform technique to obtain solutions to velocity and concentration profiles with a concluding remark that the velocity decreases due to the presence of first order chemical reaction. Chen and Arce (1997) presented the convective-diffusive mass transfer problem with

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chemical reaction in a Couette planar flow in terms of the integral-spectral methods. They discussed the effects of relevant parameters for the system on the computational algorithm with respect to convergence and numerical stability characteristics. Also, the chemical reaction effect on heat and mass transfer of an MHD fluid flow past a semi-infinite plate was analyzed by Anjali Devi and Kandasamy (2001). They solved the non-linear governing equation using Runge-Kutta Gill's method and concluded that in the presence of chemical reaction, the velocity and concentration decrease with increase of the chemical reaction parameter. The velocity also decreases and the concentration is uniform with increase of the magnetic parameter.

Muthucumaraswamy and Ganesan (2001a) investigated the effect of chemical reaction and injection as well as flow characteristics in an unsteady upward motion of an isothermal plate with mass flux through the plates. They showed that the fluid velocity decreases with increasing chemical reaction parameter. They (2001b) further examined numerically the transient natural convection flow of an incompressible viscous fluid past an impulsive started semi-infinite isothermal vertical plate with mass diffusion and homogeneous chemical reaction of first-order. They observed that owing to the presence of the first-order chemical reaction, the velocity increases during generative chemical reaction and decreases in the case of destructive chemical reaction. Muthucumaraswamy and Meenakshisundaram (2006) studied the influence of the homogeneous chemical reaction of first order on a viscous incompressible unsteady flow past an infinite vertical oscillating plate with variable temperature and mass diffusion. Applying Laplace transform method for solving dimensionless governing equations, they observed that the velocity increases with decreasing phase angle or chemical reaction parameter.

The effect of third-order fluid on the peristaltic transport in a circular cylindrical tube was established by Hayat *et al.* (2002). They compared solutions of both analytic (perturbation) and numerical solution that could have applications to a range of peristaltic flows for variety of non-Newtonian fluids. The influence of magnetic field on transient free convection flow of an electrically conducting fluid over an impulsively started isothermal vertical plate was numerically considered by Al-Odat and Al-Azab (2007). Using an implicit finite-difference scheme of Crank-Nicolson type, they found that the velocity as well as concentration decreases with increasing chemical reaction parameter. Sajid and Hayat (2007) presented the non-similar solution to the boundary layer flow problem of a third-order fluid over a stretching surface. They obtained the analytical solution of the problem using homotopy analysis method and compared their solutions with the existing results in the literature.

Stepanak and Achwal (2009) analyzed the problem of absorption with first-order chemical reaction into a liquid film in laminar flow. The results obtained in terms of sum of a hyperbolic function and infinite series of hypergeometric function showed that the length of the entrance region could vary substantially depending upon the reaction rate constant. El-Arabawy (2009) examined and made an analysis to the effects of suction/injection and chemical reaction on mass transfer characteristics over a stretching surface. He obtained analytical solutions and determined variations of embedded parameters on the problem. Also, a numerical solution attempt for unsteady convective flow of a third grade fluid past an infinite vertical porous plate was made by Nayak *et al.* (2012). They studied the effects of physical parameters on the flow and concluded that the velocity increases with an increase of the second grade elastic parameter whereas the velocity decreases with an increase of third grade elastic parameter.

Hayat *et al.* (2011) investigated the effect on mass transfer with higher order chemical reaction at a time-dependent three-dimensional boundary layer flow of an elasticoviscous fluid over a stretching surface. They computed the results by using the homotopy analysis method (HAM) and variations of embedding parameters on the velocity and concentration were graphically discussed. Other works of Hayat *et al.* (2010a, 2010b) on flow and mass transfer focused respectively on the homotopy solution for unsteady three-dimensional MHD flow over a porous space and similar solution of stretching flow with mass transfer. The boundary flow and mass transfer of a viscoelastic fluid immersed in a porous medium over a stretching surface in the presence of surface slip, chemical reaction and variable viscosity had been reported by Mahmoud (2010). Solving the problem numerically by means of the fourth order Runge-Kutta integration scheme coupled with the shooting technique, he examined the effects of various involved parameters on the velocity and concentration fields as well as the local skin-friction coefficient and local Sherwood number. Also, Raptis *et al.* (2011) studied the steady two-dimensional free-convection and mass transfer flow of an incompressible viscous fluid through a porous medium bounded by an infinite vertical limiting surface. They discussed the rate of heat transfer when the surface is subjected to a constant suction velocity. Elbashbeshy *et al.* (2011) examined the effects of suction/injection and variable chemical reaction on mass transfer characteristics over unsteady stretching surface embedded in a porous medium. They established that the velocity as well as concentration decreases with increasing suction/injection and unsteadiness parameters while the concentration decreases with increasing chemical reaction parameter. Okoya (2008) investigated the thermal transition of a reactive flow of a third-grade fluid with viscous heating and chemical reaction between two horizontal flat plates in the presence of imposed pressure gradient. He focused on the disappearance of criticality of the solution set for various parameters.

Okoya (2011) also considered how a model for the motion with exponential viscosity of a third grade fluid flowing between parallel plates affects the fully developed and laminar reactive flow. He further discussed the criticality and disappearance of criticality of physical parameter with other competing parameters. Das *et al.* (2011) studied the effects of mass transfer on an unsteady free convective flow past a vertical porous plate in a porous medium with constant suction. In their work, they also considered the interaction of an electrically conducting fluid flow with magnetic field and heat source. They established that a growing Hartmann number decreases the transient temperature of the flow fields whereas a growing permeability or heat source parameter reverses the effect. Das *et al.* (2012) further provided solutions to the problem of an unsteady hydromagnetic convective flow past an infinite vertical porous flat plate in a porous medium. Employing multi-parameter perturbation technique to obtain solutions for velocity and temperature profiles, they concluded that the permeability enhances the skin-friction as well as the rate of heat transfer at the wall.

Toki and Tokis (2007) gave exact solutions for the unsteady free convection flows on a porous plate with time-dependent heating. Keimanesh *et al.* (2011) studied the flow of a third grade non-Newtonian fluid between two parallel plates. They used the multi-step differential transform method to obtain the solution to the problem. Khan *et al.* (2014) investigated the effects of an arbitrary wall shear stress on unsteady magnetohydrodynamic conjugate flow in a porous medium with ramped wall temperature. They concluded that velocity solutions are more general and could produce a huge number of exact solutions correlative to various fluid motions. Ahmed *et al.* (2014) examined the transport modelling for magnetohydrodynamic unsteady flow over a vertical plate in Darcian porous media. They employed the finite difference approach for the solutions of the unsteady flow.

The aim of the present paper is to investigate the effect of mass transfer on transient free convective flow of an incompressible third grade fluid past a vertical porous plate in the presence of a first-order chemical reaction and suction. The work exhibits velocity distributions when a vertically upward plate suddenly sets in motion in its own plane and species concentration distributions of the fluid. The investigation is based on time-based boundary layer dimensionless variables. The numerical solutions are then obtained using Crank-Nicolson finite difference scheme with modified Newton's iteration technique. Results are presented graphically and discussed quantitatively for parameter of practical interest from physical point of view.

Mathematical Formulation

Consider a transient, laminar, incompressible and viscoelastic third grade fluid flow with mass transfer past an infinite vertical porous plate in the presence of a first-order chemical reaction. The fluid properties are assumed to be constant in the limited temperature range. The x' - axis is taken along the plate vertically upwards and y' - axis is perpendicular to it. The plate is set in motion in its own plane with a velocity *U(t)*. The concentration of diffusing species is very small in comparison to the other chemical species. The species concentration far away from the plate C_∞ is infinitesimally small. The chemical reactions are taking place in the flow regime, and all other physical properties are assumed to be constant. Since the plate is infinitely long, the physical variables are functions of y' and t' only. Hence, from the continuity equation, the velocity field is obtained as:

$$u' = u'(y', t'),\ v' = -V_0 \qquad (1)$$

where u' and v' are the velocities of the fluid along x' and y' axes respectively, and $V_0 > 0$ indicates suction velocity.

Flow Analysis for Mass Transfer

The constitutive equation for Cauchy's stress tensor of an incompressible homogeneous third grade fluid, according to Rivlin and Ericksen (1955), Coleman and Noll (1965) and Truesdell and Noll (2004), is:

$$\tau = -p_1 I + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 \left(trA_1^2\right) A_1 \qquad (2)$$

where $-p_1 I$ is the spherical part of the stress due to the constraint of incompressibility; p_1 is the pressure; I is an identity tensor, $\propto_1$ and $\propto_2$ are normal stresses, and β_1, β_2 and β_3 are material constants. The Rivlin-Ericksen tensors A_1, A_2

$$A_1 = (gradV) + (gradV)^T$$

$$A_{n+1} = \left(\frac{\partial}{\partial t} + V \cdot \nabla\right) A_n + (gradV)^T A_n + A_n (gradV),\ n = 1,2 \qquad (3)$$

where Δ is the gradient operator, $\frac{d}{dt}$ is the material time derivative and V is the velocity.

If all the motions of the fluid are to be compatible with thermodynamics in the sense that these motions meet the Clausius-Duhem inequality and if it is assumed that the specific Helmholtz free energy is a minimum when the fluid is locally at rest, then

$$\mu_1 \geq 0, \qquad \alpha_1 \geq 0,$$

$$|\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3},\ \beta_1 = \beta_2 = 0,\ \beta_3 \geq 0. \qquad (4)$$

For a detailed thermodynamic analysis on the model represented by equation (2), one can refer to the work of Fosdick and Rajagopal (1980). Therefore, the constitutive relation for a thermodynamically compatible third grade fluid becomes:

$$\tau = -pI + \mu_1 A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_3 \left(trA_1^2\right) A_1 \qquad (5)$$

The stress components of (2) by virtue of equation (1) are:

$$\tau_{x'x'} = -p + \alpha_2 \left(\frac{\partial u'}{\partial y'}\right)^2 + 2\beta_2 \frac{\partial u'}{\partial y'} \frac{\partial^2 u'}{\partial y' \partial t'}, \qquad (6)$$

$$\tau_{y'y'} = -p + (2\alpha_1 + \alpha_2)\left(\frac{\partial u'}{\partial y'}\right)^2 + (6\beta_1 + 2\beta_2)\frac{\partial u'}{\partial y'} \frac{\partial^2 u'}{\partial y' \partial t'}, \qquad (7)$$

$$\tau_{z'z'} = -p \,, \tag{8}$$

$$\tau_{x'y'} = \mu\frac{\partial u'}{\partial y'} - \alpha_1 V_0 \frac{\partial^2 u'}{\partial y'^2} + \alpha_1 \frac{\partial^2 u'}{\partial y' \partial t'} + 2(\beta_2 + \beta_3)\left(\frac{\partial u'}{\partial y'}\right)^3 + \beta_1\left(\frac{\partial^3 u'}{\partial y' \partial t'^2}\right) \tag{9}$$

$$\tau_{x'z'} = \tau_{z'y'} = 0 \tag{10}$$

where $\tau_{x'y'} = \tau_{y'x'}$, $\tau_{x'z'} = \tau_{z'x'}$, $\tau_{y'z'} = \tau_{z'y'}$

Inserting the stress components (6-10) and velocity field given by (1) and using (5) in the equation of motion while neglecting body force, we have:

$$\rho\left(\frac{\partial u'}{\partial t'} - V_0 \frac{\partial u'}{\partial y'}\right) = \mu \frac{\partial^2 u'}{\partial y'^2} + \alpha_1\left(\frac{\partial^3 u'}{\partial y'^2 \partial t'} - V_0 \frac{\partial^3 u'}{\partial y'^3}\right) + 6\beta_3\left(\frac{\partial u'}{\partial y'}\right)^2 \frac{\partial^2 u'}{\partial y'^2} + \rho g' \beta_C (C' - C'_\infty) \tag{11}$$

Mass Transfer Analysis

For the thermodynamically compatible third grade fluid using (1), the species concentration equation with first-order homogeneous chemical reactions for the flow field is:

$$\frac{\partial C'}{\partial t'} - V_0 \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - k_1 (C' - C'_\infty) \tag{12}$$

where D and k_1 are mass diffusivity and rate of chemical reaction respectively.

The initial and boundary conditions are:

$$t' \le 0 : u' = 0, C' = 0, \forall y > 0 \tag{13}$$

$$t' > 0 : u' = U(t') = \frac{U^{2n+1}}{v^n} e^{a't'} t'^n, \quad when \quad y' = 0 \tag{14}$$

$$C' = C'_\infty + (C'_w - C'_\infty) = 1, \quad when \quad y' = 0 \tag{15}$$

$$u' = 0, \quad when \quad y \to \infty, \ \frac{\partial u'}{\partial y'} = 0, \ when \ y \to \infty \tag{16}$$

$$C' = C'_\infty, \ when \ y' \to \infty \tag{17}$$

The following dimensionless variables are introduced:

$$u = \frac{u'}{U}, \qquad y = \frac{y'U}{\nu}, \qquad a = \frac{a'\nu}{U^2}, \qquad t = \frac{t'U^2}{\nu} \tag{18}$$

Applying the similarity transformation for scaling species concentration equations in mass transfer analysis where the species concentration, $\phi = \dfrac{C' - C'_\infty}{C'_w - C'_\infty}$ to obtain:

$$\frac{\partial u}{\partial t} - \omega \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \alpha\left(\frac{\partial^3 u}{\partial y^2 \partial t} - \omega \frac{\partial^3 u}{\partial y^3}\right) + \beta\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + Gc\phi \tag{19}$$

$$\frac{\partial \phi}{\partial t} = \omega \frac{\partial \phi}{\partial y} + \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - \gamma \phi \tag{20}$$

where $\omega = \frac{V_0}{U}$, $\gamma = \frac{k_1 \nu}{U^2}$, $\alpha = \frac{\alpha_1 U^2}{\rho \nu^2}$, $\beta = \frac{6\beta_3 U^4}{\rho \nu^3}$, $Gc = \frac{\beta_C g' \nu (C'_w - C'_\infty)}{U^3}$, $Sc = \frac{D}{\nu}$

are suction, rate of chemical reaction, elastic second and third grade parameters, solutal Grashof and Schmidt numbers respectively.

The boundary conditions (13) - (17) are also non-dimensionalized using (18) with the dimensionless variables $\omega = \frac{\omega' \nu}{U^2}$ to obtain the following initial and boundary conditions:

$$t \le 0 : y = 0, u = 0, \phi = 0; \tag{21}$$

$$t > 0 : u = e^{at} t^n, \ when \ y = 0; \tag{22}$$

$$\phi = 1, \ when \quad y = 0; \tag{23}$$

$$u \to 0, \ when \ y \to \infty, \ \frac{\partial u}{\partial y} \to 0, \ when \ y \to \infty; \tag{24}$$

$$\phi \to 0, \ when \ y \to \infty; \tag{25}$$

Computational Procedure

The governing nonlinear coupled partial differential equations (19) and (20) with the initial and boundary conditions (21-25) are solved using Crank-Nicolson finite difference scheme, which has been discussed by Conte and De Boor (1980), Jain *et al*. (1999), Ganesan and Palani (2002) and Baoku *et al*. (2012). Therefore, the governing equations based on the transient state conditions are discretized. The numerical method of Crank-Nicolson type does not restrict the value of r to be chosen. The finite difference equations corresponding to the governing equations are given as:

$$
\begin{aligned}
u_{i,j+1}-u_{i,j} &= \left(\frac{\omega rh}{4}+\frac{r}{2}+\frac{\alpha}{h^2}-\frac{\omega\alpha r}{h}\right)u_{i+1,j+1}+\left(\frac{r}{2}-\frac{\omega rh}{4}+\frac{\alpha}{h^2}-\frac{\omega\alpha r}{h}\right)u_{i-1,j+1} \\
&+\left(\frac{\omega rh}{4}+\frac{r}{2}\right)u_{i+1,j}+\left(\frac{r}{2}-\frac{\omega rh}{4}+\frac{\alpha}{h^2}\right)u_{i-1,j}-2\left(\frac{r}{2}+\frac{\alpha}{h^2}\right)u_{i+1,j}-2\left(\frac{r}{2}+\frac{\alpha}{h^2}\right)u_{i,j} \\
&+\frac{\alpha}{h^2}u_{i+1,j}-\frac{\omega\alpha r}{2h}\left[\left(-u_{i-2,j+1}+u_{i+2,j+1}\right)-\left(-u_{i-2,j}+2u_{i-1,j}-2u_{i+1,j}+u_{i+2,j}\right)\right] \\
&+\frac{\beta r}{32h}\begin{bmatrix}\left(u_{i+1,j+1}\right)^2+\left(u_{i-1,j-1}\right)^2+\left(u_{i+1,j}\right)^2+\left(u_{i-1,j}\right)^2-2u_{i+1,j+1}u_{i-1,j-1} \\ +2u_{i+1,j+1}u_{i+1,j}-2u_{i+1,j+1}u_{i-1,j}-2u_{i+1,j}u_{i-1,j-1}+2u_{i-1,j}u_{i-1,j-1}-2u_{i+1,j}u_{i-1,j}\end{bmatrix} \\
&\left(u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}+u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right)+2rh^3Gc\left(\frac{\phi_{i,j+1}+\phi_{i,j}}{2}\right)
\end{aligned}
\tag{27}
$$

$$
\phi_{i,j+1}=\left(\frac{\omega Sch\,r+2r}{2Sc}\right)\phi_{i+1,j+1}+\left(\frac{2r-\omega Sch\,r}{2Sc}\right)\phi_{i-1,j+1}-\left(\frac{4r+\gamma Sch^2 r}{2Sc}\right)\phi_{i,j+1}+\left(\frac{2-\gamma h^2 r}{2}\right)\phi_{i,j}
\tag{28}
$$

where i designates the grid point along y-direction, j along t-direction and $r=\Delta t/h^2$. Hence, the equations of motion and species concentration are reduced to a system of algebraic nonlinear coupled-equations. The mesh size h is 0.05 with time step t =0.1. The values of $u\ (y,t)$ and $\phi\ (y,t)$ are known for all grid points when $t=0$ from the initial conditions. Modified Newton's iterative technique is employed to solve the system of coupled nonlinear algebraic equations. Computations are carried out by moving along y-direction. After computing values corresponding to each i at a time level, the values at the next time level are determined in the similar manner.

The implicit nature of Crank-Nicolson method is unconditionally stable and has local truncation error $O\left[(\Delta t)^2, h^2\right]$ which tends to zero as Δt and h^2 tend to zero. There is no drawback of conditional stability from one level to the next. The implicit method gives stable solutions and requires the iterative procedures which were done at each step forward in time because this problem is an initial-boundary value problem with a finite number of spatial grid points. Though, the corresponding difference equations do not automatically guarantee the convergence of the mesh $h \to 0$. To achieve maximum numerical efficiency, the tridiagonal and pentadiagonal procedures were used to solve the two-point conditions for (20) and four-point conditions for (19) respectively. The above procedure was transformed into Maple code as described by Heck (2003). The convergence of the process was quite satisfactory and the numerical stability of the method was guaranteed by the implicit nature of the scheme. Hence, the scheme is consistent; stability and consistency ensure convergence.

Results and Discussion

The effect of transient free convection flow on mass transfer of an incompressible third grade fluid past a vertical porous plate in the presence of a first-order chemical reaction was investigated. The research exhibits velocity distributions when a vertically upward plate suddenly sets in motion at a velocity U in its own plane and concentration distributions. The governing equations of the flow field are solved by employing an efficient Crank-Nicolson finite difference technique with modified Newton's iterative technique and approximate solutions are obtained for velocity and species concentration distributions. The influences of some controlling parameters on the flow field are analyzed and discussed with the aid of velocity profiles (Figures 1-3) and species concentration profiles (Figures 4 and 5).

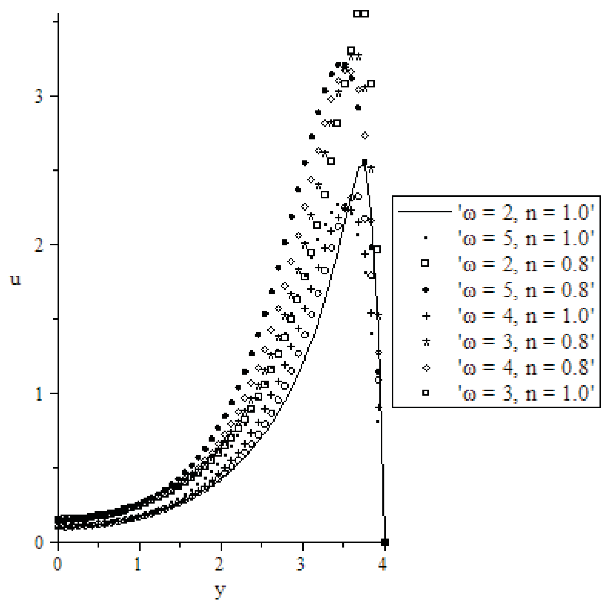

Figure 1: Velocity distribution when n=0.8 and n=1.0 for values of ω at β=5.0 , α=2.0, Gc=10 .

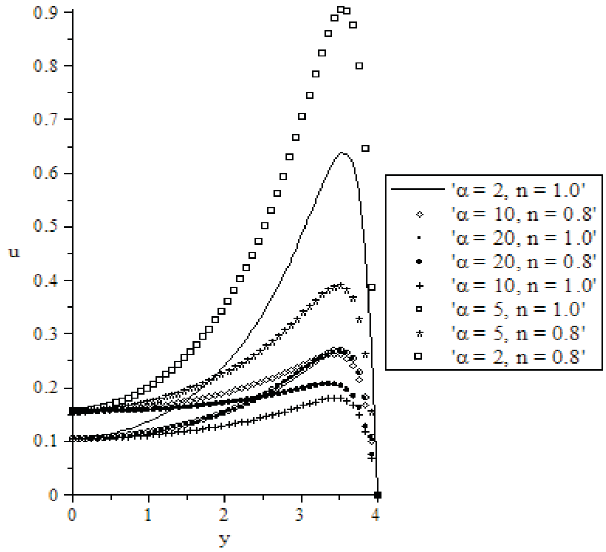

Figure 2: Velocity distribution when n=0.8 and n =1.0 for values of α at ω=3.0 , β=5.0, Gc=10 .

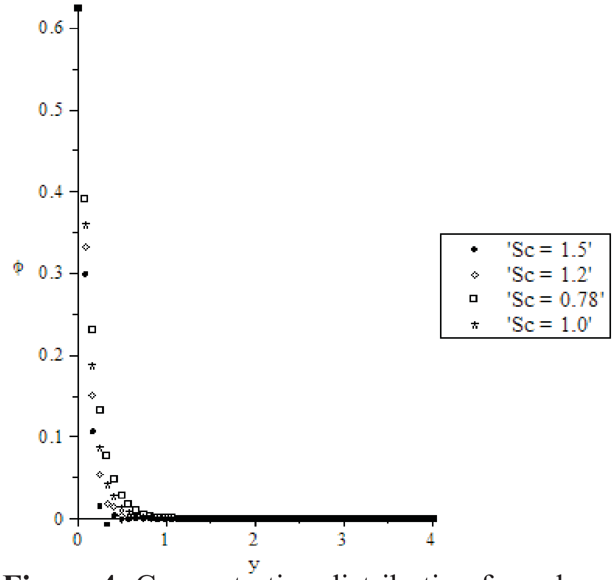

Figure 4: Concentration distribution for values of Sc at ω =3.0 and γ =12 .

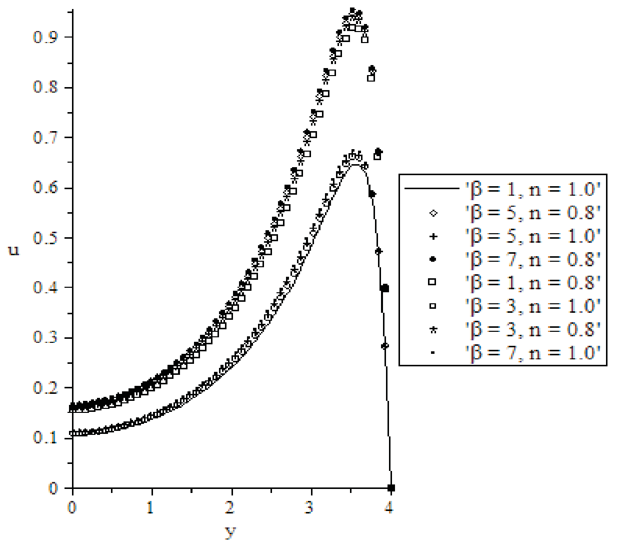

Figure 3: Velocity distribution when n=0.8 and n =1.0 for values of β at ω=3.0, α=2.0, Gc=10 .

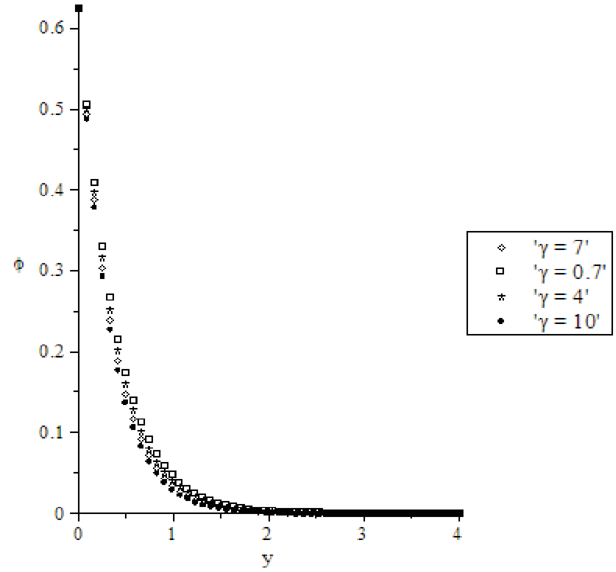

Figure 5: Concentration distribution for values of γ at Sc =7.0 and ω =3.0 .

During numerical calculations, the values of the some parameters have been realistically chosen while the values of other parameters are chosen arbitrarily. The effect of the suction parameter on the transient velocity distributions are shown in Figure 1. It is observed from Figure 1 that the velocity fluid decreases with the increase in suction parameter w for both cases n=1 and n = 0.8 , that is constant and variable acceleration respectively, at any point within the fluid. Figure 2 displays the influence of the elastic parameter β on the velocity distribution. It is clear from Figure 2 that the fluid velocity increases for both constant and variable accelerations when the third grade parameter β increases. However, the reverse is the case for the second grade parameters. This observation is similar to that made by Nayak *et al.* (2012). Figure 3 analyzed the effect of mass diffusivity shown by Schmidt number Sc on the velocity distribution for both constant and variable accelerations along the vertical plate. It is obvious from Figure 3 that an increase in Sc leads to a rise to the velocity of any point within the fluid.

The species concentration of the flow field suffers a substantial change with the variation of the flow parameters such as the Schmidt number Sc and the rate of chemical reaction parameter γ. It is observed in Figure 4 that when there is an increase in Schmidt number, there is a reduction in the concentration field. Finally, it is evident from Figure 5 where destructive chemical reactions are considered, that the consequence of increasing the rate of chemical reaction has the effect of decreasing the species concentration distribution.

Conclusion

The transient flow and mass transfer of a third grade fluid past a vertical porous plate in the presence of chemical reaction is studied. The governing time-based partial differential equations are solved using an efficient numerical scheme of implicit Crank-Nicolson finite difference technique with the modified Newton's iterative method. Simulation of the scheme is done with the aid of Maple codes to obtain approximate solutions of the problem. The solution procedure is valid for all values of elastic parameters unlike perturbation and power series methods that are valid for small values of the elastic parameters. The conclusions of the study are:

1. The velocity increases significantly with increasing solutal Grashof number and Schmidt number.
2. The fluid velocity also increases as the value of second grade parameter increases. However, the third grade parameter has opposite effect on the velocity.
3. An increase in the suction parameter has the influence of decreasing the velocity distribution as well as the species concentration distribution.
4. As the rate of chemical reaction and Schmidt number increase, it results into a decrease in the species concentration in all part of the fluid.

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